RBC Capital Markets

Introduction To Bond Valuation

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Introduction To Bond Valuation

- General Principles of Valuation
- Traditional Approach to Valuation
- The Arbitrage-Free Valuation Approach
- Measurement of Interest Rate Risk

General Principles of Valuation

- Fundamental Principle of Bond Valuation
  - Calculate ***Present Value*** of the Expected Cash Flows
  - Step 1: Estimate the expected cash flows
    - Cash Flow → the cash that is expected to be received in the future
    - Characteristics of Bond Cash Flows
  - Step 2: Determine the Appropriate Interest Rate or Interest Rates
    - On-the-run Treasury Yield
    - Premiums
  - Step 3:Discounting the Expected Cash Flows

$$PV = \sum_{t=1}^{n} \frac{\text{Expected Cash Flow in Period } t}{(1+i)^t}$$

General Principles of Valuation

- Application I: Single Discount Rate
  - A bond matures in four years. Its coupon is 10% and has a maturity value of 100. The bond pays coupon annually and discount rate is 8%.

$$PV = \prod_{t=1}^{n} \frac{\text{Expected Cash Flow in Period } t}{(1+i)^t}$$

Year	Cash Flow
1	$10
2	$10
3	$10
4	$110

$$\text{Year 1: present value}_1 = \frac{\$10}{(1.08)^1} = \$9.2593$$

$$\text{Year 2: present value}_2 = \frac{\$10}{(1.08)^2} = \$8.5734$$

$$\text{Year 3: present value}_3 = \frac{\$10}{(1.08)^3} = \$7.9383$$

$$\text{Year 4: present value}_4 = \frac{\$110}{(1.08)^4} = \$80.8533$$

Bond Price = $106.6243

General Principles of Valuation

- Application II: Multiple Discount Rates
  - A bond matures in four years. Its coupon is 10% and has a maturity value of 100. The bond pays coupon annually and discount rates are Y1: 6.8%, Y2: 7.2%, Y3: 7.6%, Y4: 8%

$$PV = \sum_{t=1}^{n} \frac{\text{Expected Cash Flow in Period } t}{(1+i_t)^t}$$

Year	Cash Flow
1	\$10
2	\$10
3	\$10
4	\$110

$$\text{Year 1: present value}_1 = \frac{\$10}{(1.068)^1} = \$9.3633$$

$$\text{Year 2: present value}_2 = \frac{\$10}{(1.072)^2} = \$8.7018$$

$$\text{Year 3: present value}_3 = \frac{\$10}{(1.076)^3} = \$8.0272$$

$$\text{Year 4: present value}_4 = \frac{\$110}{(1.08)^4} = \$80.8533$$

Bond Price = $106.9456

General Principles of Valuation

- Application III: Valuing a Bond Between Coupon Dates

|← Interest earned by seller →|← Interest earned by buyer →|

last coupon payment date | settlement date | next coupon payment date

$$w \text{ periods} = \frac{\text{days between settlement date and next coupon date}}{\text{days in coupon period}}$$

$$\text{present value}_t = \frac{\text{expected cash flow}}{(1+i)^{t-1+w}}$$

Suppose there are three semiannual coupon remaining for a 10% coupon bond with:

10. 78 days between the settlement date and the next coupon payment date
11. 182 days in the coupon period.

w is 0.4286 (= 78/182). The present value of each cash flow (8% annual discount rate):

$$\text{Period 1: present value}_1 = \frac{\$5}{(1.04)^{0.4286}} = \$4.9167 \qquad \text{Period 2: present value}_2 = \frac{\$5}{(1.04)^{1.4286}} = \$4.7276$$

$$\text{Period 3: present value}_3 = \frac{\$105}{(1.04)^{2.4286}} = \$95.4602$$

Bond Full Price is $105.1045

General Principles of Valuation

- Application IV: Accrued Interest and the Clean Price

|← Interest earned by seller →|← Interest earned by buyer →|

last coupon payment date — settlement date — next coupon payment date

$$w \text{ periods} = \frac{\text{days between settlement date and next coupon date}}{\text{days in coupon period}}$$

$$\text{Accrued Interst} = \text{semiannual coupon payment } (1 - w)$$

$$\text{Clean Price} = \text{Full Price} - \text{Accrued Interest}$$

Suppose there are three semiannual coupon remaining for a 10% coupon bond with:

11. 78 days between the settlement date and the next coupon payment date
12. 182 days in the coupon period.

w is 0.4286 (= 78/182). The present value of each cash flow (8% annual discount rate):

$$\text{Accrued Interest} = \$5 \text{ x } (1 - 0.4286) = \$2.8571$$

$$\text{Clean Price} = \text{Full Price} - \text{Accrued Interest} = \$105.1045 - \$2.8571 = \$102.2474$$

General Principles of Valuation

- Day Count Conventions

|← Interest earned by seller →|← Interest earned by buyer →|

last coupon payment date | settlement date | next coupon payment date

$$\text{Accrued Interst} = \text{semiannual coupon payment x } \frac{\text{days in AI period}}{\text{days in coupon period}}$$

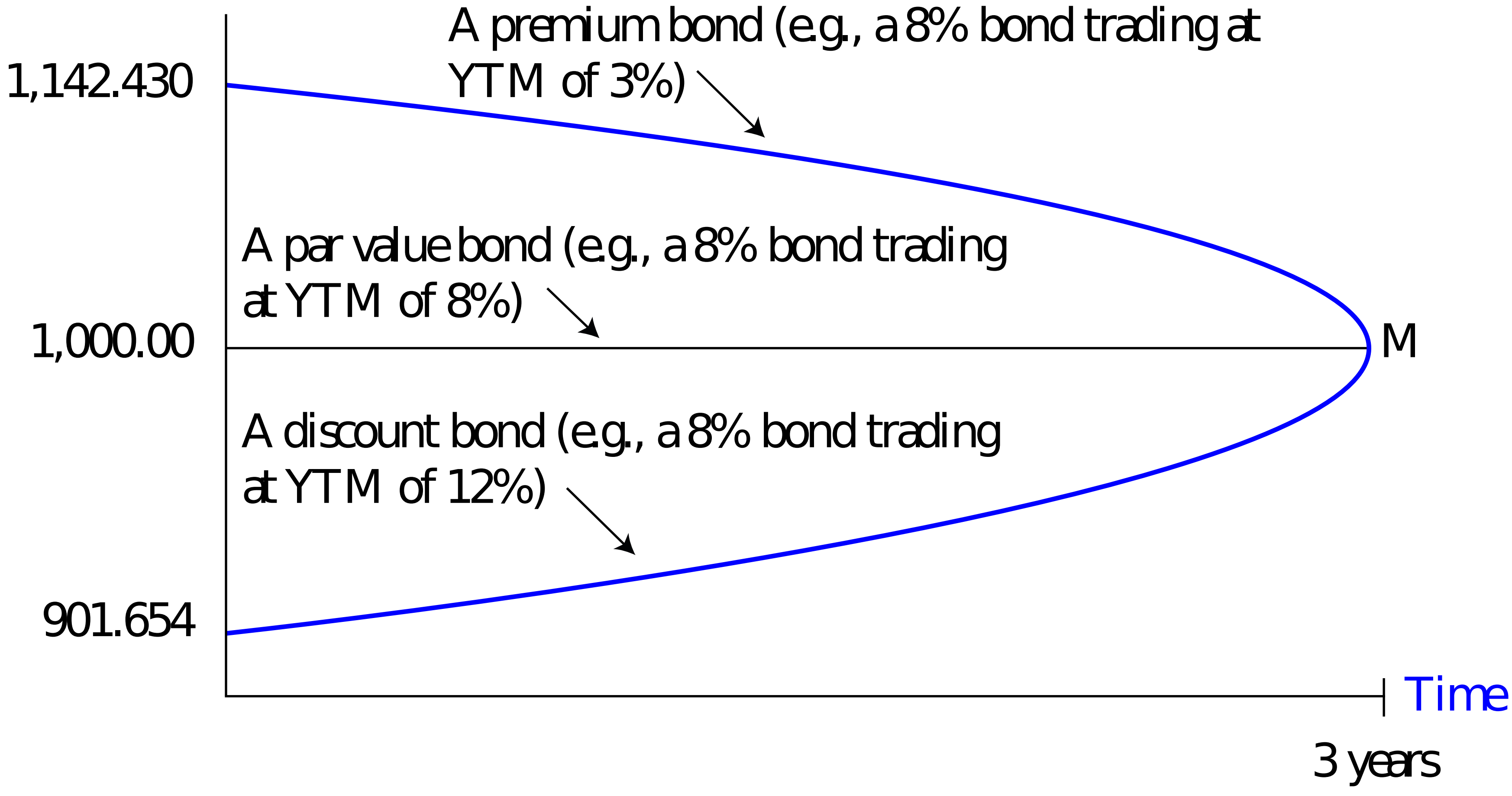
<u>ACT/365</u>

<u>ACT/ACT</u>

<u>30/360</u>

General Principles of Valuation

- Price Change as Maturity Approaches



Traditional Approach to Valuation

	Coupon Rate		
Period	12%	8%	0%
1-19	$6	$4	$0
20	$106	$104	$100

- To discount every cash flow of a bond by the same interest rate (or discount rate).
- The rate used is the yield for the on-the-run issue obtained from the Treasury yield curve.

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Arbitrage-Free Valuation Approach

	Coupon Rate		
Period	12%	8%	0%
1-19	$6	$4	$0
20	$106	$104	$100

- To view 10 year 8% coupon Treasury issue as a package of zero-coupon bonds whose maturity value is the amount of the cash flow and whose maturity date is the date that the cash flow is received
- Arbitrage-free value of a bond is the total value of the individual zero-coupon bonds discounted based on Spot Rate Curve
- Arbitrage opportunity: the market price of the bond and the price of the package of zero-coupon bonds are not the same

Arbitrage-Free Valuation Approach

- Application I: Determination of the Arbitrage-Free Value of the 8% 10-Year Treasury

Period	Years	Cash Flow ($)	Spot Rate (%)	PV ($)
1	0.5	4	3.0000	3.9409
2	1.0	4	3.3000	3.8712
3	1.5	4	3.5053	3.7968
4	2.0	4	3.9164	3.7014
5	2.5	4	4.4376	3.5843
6	3.0	4	4.7520	3.4743
7	3.5	4	4.9622	3.3694
8	4.0	4	5.0650	3.2747
9	4.5	4	5.1701	3.1791
10	5.0	4	5.2772	3.0829
11	5.5	4	5.3864	2.9861
12	6.0	4	5.4976	2.8889
13	6.5	4	5.6108	2.7916
14	7.0	4	5.6643	2.7055
15	7.5	4	5.7193	2.6205
16	8.0	4	5.7755	2.5365
17	8.5	4	5.8331	2.4536
18	9.0	4	5.9584	2.3581
19	9.5	4	6.0863	2.2631
20	10.0	104	6.2169	56.3830
			Total	115.2621

- What would happen if the market price of the bond is at $114.8775?
- What would happen if the market price of the bond is at $115.9145?

Arbitrage-Free Valuation Approach

- Application II: Seeking Arbitrage Profit from Stripping the 8% 10-Year Treasury

Period	Years	Sell for	Buy for	Arbitrage Profit
1	0.5	3.9409	3.8835	0.0574
2	1.0	3.8712	3.7704	0.1008
3	1.5	3.7968	3.6606	0.1362
4	2.0	3.7014	3.5539	0.1475
5	2.5	3.5843	3.4504	0.1339
6	3.0	3.4743	3.3499	0.1244
7	3.5	3.3694	3.2524	0.1170
8	4.0	3.2747	3.1576	0.1171
9	4.5	3.1791	3.0657	0.1134
10	5.0	3.0829	2.9764	0.1065
11	5.5	2.9861	2.8897	0.0964
12	6.0	2.8889	2.8055	0.0834
13	6.5	2.7916	2.7238	0.0678
14	7.0	2.7055	2.6445	0.0610
15	7.5	2.6205	2.5674	0.0531
16	8.0	2.5365	2.4927	0.0438
17	8.5	2.4536	2.4201	0.0335
18	9.0	2.3581	2.3496	0.0085
19	9.5	2.2631	2.2811	-0.0180
20	10.0	56.3830	57.5823	-1.1993
		115.2621	114.8775	0.3846

Arbitrage-Free Valuation Approach

- Application III: Arbitrage Profit from Reconstituting back the 8% 10-Year Treasury

Period	Years	Sell for	Buy for	Arbitrage Profit
1	0.5	3.8859	3.9409	-0.0550
2	1.0	3.7750	3.8712	-0.0962
3	1.5	3.6673	3.7968	-0.1295
4	2.0	3.5627	3.7014	-0.1387
5	2.5	3.4611	3.5843	-0.1232
6	3.0	3.3623	3.4743	-0.1120
7	3.5	3.2664	3.3694	-0.1030
8	4.0	3.1732	3.2747	-0.1014
9	4.5	3.0827	3.1791	-0.0964
10	5.0	2.9948	3.0829	-0.0881
11	5.5	2.9093	2.9861	-0.0767
12	6.0	2.8263	2.8889	-0.0626
13	6.5	2.7457	2.7916	-0.0459
14	7.0	2.6674	2.7055	-0.0382
15	7.5	2.5913	2.6205	-0.0292
16	8.0	2.5174	2.5365	-0.0192
17	8.5	2.4455	2.4536	-0.0081
18	9.0	2.3758	2.3581	0.0176
19	9.5	2.3080	2.2631	0.0449
20	10.0	58.2962	56.3830	1.9132
		115.9145	115.2621	0.6524

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Measurement of Interest Rate

- Full Valuation Approach
  - Re-value the bond position or a portfolio when interest rates change
  - Scenario (Stress) Testing:
    - For one bond position:
      - Project market rate changes of +/- 5bp, +/- 10bp, +/- 25bp, ….
      - Re-Calculate bond position for each projected rate change
    - For bond portfolio:
      - Project market rates change scenario
      - Re-calculate portfolio position

Measurement of Interest Rate

- Duration/Convexity Approach
  - Duration is the approximate percentage change in value for a 100 basis point (bp) change in rates
  - Duration calculation:

$$\text{duration} = \frac{V_{-} - V_{+}}{2(V_0)(\Delta y)}$$

  - Example: 8% coupon 15-year option-free bond selling at $100.00 to yield 8%. If the yield shocked up and down by 50 bp, the duration calculation is as follow:

$$V_{-} = 104.414$$

$$V_{+} = 95.848$$

$$V_0 = 100$$

$$\Delta y = 0.005$$

$$\text{duration} = \frac{104.414 - 95.848}{2 \times (100)(0.005)} = 8.57$$

Measurement of Interest Rate

- Duration/Convexity Approach
  - Application: Duration Effect

$$\text{duration effect} = -D \times \Delta y$$

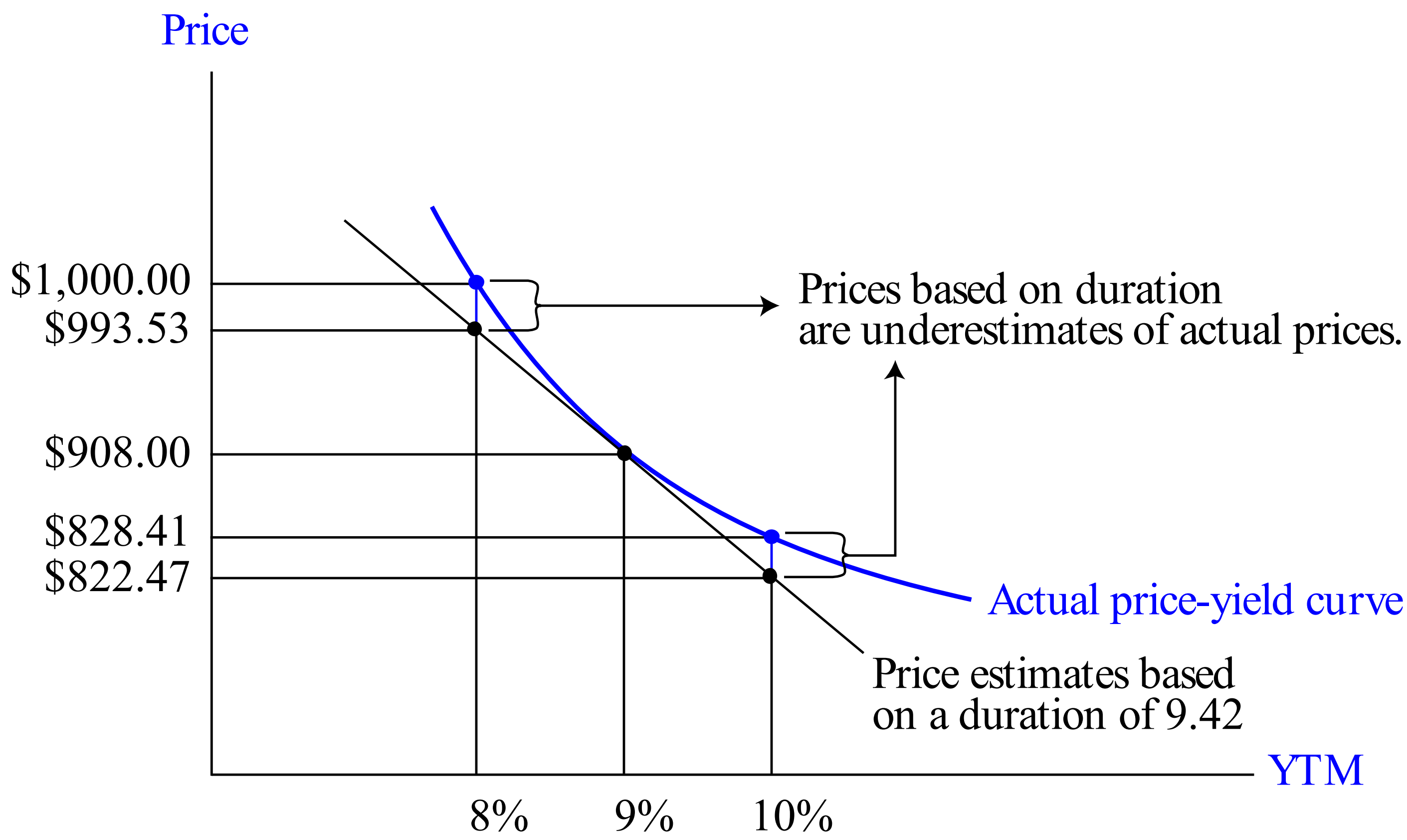
  - Example: Let's use the same 8% 15-year par bond which has a duration of 8.57. If YTM *increases* 0.3% or 30bp, bond price *decreases* by *approximately*:

$$-8.57 \times 0.3\% = -2.57\%$$

  - Duration Measures
    - Macaulay Duration
    - Modified Duration
    - Effective Duration

Measurement of Interest Rate

- Duration/Convexity Approach
  - Convexity Adjustment: Duration based estimates of new bond prices are below actual prices for option-free bonds



Measurement of Interest Rate

- Duration/Convexity Approach
  - Convexity Calculation:

$$\text{convexity measure} = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2}$$

  - Convexity Adjustment:

$$\text{convexity adjustment} = \text{convexity} \times (\Delta \text{y})^2$$

  - Example: Recall our 8% 15-year par bond with duration = 8.57. For a 50bp change in yield, the convexity is calculated as follow:

$$\text{convexity} = \frac{104.414 + 95.848 - 2 \times 100}{2 \times 100 \times 0.005^2} = 52.4$$

$$\text{convexity adjustment} = 52.4 \times (0.005)^2 = 0.00131 \text{-->} 0.131\%$$

  - Approximate price change in percentage by both duration and convexity for -50bp and +50 changes:

$$\text{approximate price change\%} = \text{duration effect} + \text{convexity adjustment}$$

$$\text{app. price change\% (- 50bp)} = -8.57\,(-0.005) + 52.4\,(-0.005)^2 = +4.416\%$$

$$\text{app.price change\% (+ 50bp)} = -8.57\,(+0.005) + 52.4\,(+0.005)^2 = -4.154\%$$

Measurement of Interest Rate

- Price Value of a Basis Point (PVBP or DV01)
  - The absolute value of the change in the price of a bond for a 1 basis point change in yield:

    PVBP = | initial price – price if yield is changed by 1 bp |

  - Example: Using the same bond of 8% 15-year selling at par. If the yield decreases by 1bp, the price is 100.085647, we have

    PVBP = | 100.00 – 100.085647 | = 0.085647